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Eccentric Connectivity and Connective Eccentricity Indices of Generalized Petersen Graphs

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Abstract

Research on the topological indices based on eccentricity of vertices of a molecular graph has been intensively rising recently. The eccentric connectivity index and the connective eccentricity index are belonging to this class of indices. In this paper we computed the exact value of the eccentric connectivity and the connective eccentricity indices of generalized Petersen graphs.

Keywords: Connective eccentric index, Eccentric connectivity index, Generalized Petersen graphs.

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1. Introduction

Chemical graph theory has an important effects to develop new drugs in medicine and pharmacology by using topological indices. Eccentric connectivity index and connective eccentricity index are among these class of indices. Sharma et al. [1] introduced the eccentric connectivity index, for a graph, which has been employed successfully for the development of numerous mathematical models for the prediction of biological activities of diverse nature [2, 3]. Wang [5] investigated extremal trees of the eccentric connectivity index. Venkatakrishnan et al. [6] computed the eccentric connectivity index of generalized thorn graphs.Das and Arani [7] compared between the Szeged index and the eccentric connectivity index. Ashrafi et al. [8] computed the exact value of the eccentric connectivity index of nanotubes and nanotori. Morgan et al. [9], investigated the extremal regular graphs with respect to the eccentric connectivity index. Doslić and Saheli [10] studied the eccentric connectivity index of composite graphs. Zhang et al. [11] characterized maximal graphs respect to eccentric connectivity index. Dankelmann et al.[12] studied the relation between Wiener index and eccentric connectivity index. Eskender and Vumar [13] computed the exact value of eccentric connectivity index and eccentric distance sum of some graph operations. Hua and Das [14] studied the relationship between the eccentric connectivity index and Zagreb indices. Zhang et al. [15] investigated the minimal eccentric connectivity indices of graphs. For more detailed discussion we refer the reader to [16] and references therein. Gupta et al. [4] introduced connective eccentricity index

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when investigating the antihypertensive activity of derivatives of N-benzylimidazole. Yu and Feng [17] derived upper or lower bounds for the connective eccentricity index in terms of some graph invariants such as the radius, independence number, vertex connectivity, minimum degree, maximum degree etc. Moreover, the authors in [17] investigated the maximal and the minimal values of connective eccentricity index among all n-vertex graphs with fixed number of pendent vertices, characterized the extremal graphs and studied the cactus on n vertices with k cycles having the maximal connective eccentricity index. Yu et al. [18] studied the connective eccentricity index of trees and unicyclic graphs with given diameter. Xu et al. [19] investigated some extremal results on connective eccentricity index. For more detailed discussion we refer the interested reader to [20] and references therein. Computing some graph invariants of generalized Petersen graphs have been well studied in graph theory. For coloring of generalized Petersen graphs see in [21] and references therein. For computing domination number of generalized Petersen graphs see in [22] and references therein. For computing labeling of generalized Petersen graphs see in [23] and references therein. For computing decycling number of generalized Petersen graphs see in [24] and references therein. And for computing connectivity of generalized Petersen graphs see in [25] and references therein. There is not any study related to computing topological index of generalized Petersen graphs in the literature for the time being. The aim of this paper is to compute the eccentric connectivity and the connective eccentricity indices of generalized Petersen graphs.

2. Preliminaries

Let G be a connected simple graph with the vertex set V and the edge set E. The distance between two vertices u, v of G, written $d(u, v)$, is the length of a shortest u–v path in G. For any vertex v of G, the eccentricity of v, denoted by ϵ_v, is the largest distance from v to other vertices in G. If the vertex u is the furthest vertex from v, then the u-v path is called the eccentric path of v. The degree of any vertex v of G, denoted by d_v, is the number of edges incident to v. If all degrees are r in G, then G is called r-regular graph. The eccentric connectivity index of a simple connected graph defined as;

$$\varepsilon^c(G) = \sum_{v \in V} \varepsilon_v d_v. \tag{2.1}$$

And the connective eccentricity index of a simple connected graph defined as;

$$\varepsilon^{ce}(G) = \sum_{v \in V} \frac{d_v}{\varepsilon_v}. \tag{2.2}$$

The generalized Petersen graph $GP(n, k)$ is the graph with vertex set $V = U \cup W$, where $U = \{u_i : 0 \leqslant i \leqslant n-1\}$ and $W = \{w_i : 0 \leqslant i \leqslant n-1\}$ and the edge set is the form of

$$E = \{u_i u_{i+1}, u_i w_i, w_i w_{i+k} : 0 \leqslant i \leqslant n-1, \text{subscripts modulo n}$$

3. Main Results

In this section, firstly we compute the eccentricities of the vertices U and W of $P(n, 1)$ and $GP(n, 2)$ and secondly, we compute the eccentric connectivity index and the connective eccentricity index of generalized Petersen graphs $GP(n, 1)$ and $GP(n, 2)$.

Proposition 3.1. *The eccentricitiy of a vertex u_i of U in $GP(n, 1)$ is $\varepsilon_{u_i} = \lceil \frac{n+1}{2} \rceil$.*

Proof. Notice that the generalized Petersen graph $GP(n, 1)$ consists of two n-vertex cycles in which corresponding vertices of U and W say u_i and w_i are adjacent to each other. And we know that the eccentricity of a vertex of a n-vertex cycle equals $\lfloor \frac{n}{2} \rfloor$. Therefore from this point of view, we can say that the eccentric vertex of a vertex from U lies in the set W. Let $u_i \in U$. Then the path

$$u_i w_i w_{i+1} w_{i+2} ... w_{i+\lfloor \frac{n}{2} \rfloor - 1} w_{i+\lfloor \frac{n}{2} \rfloor}$$

or

$$u_i u_{i+1} u_{i+2} ... u_{i+\lfloor \frac{n}{2} \rfloor - 1} u_{i+\lfloor \frac{n}{2} \rfloor} w_{i+\lfloor \frac{n}{2} \rfloor}$$

is the eccentric path of u_i. Thus $\varepsilon_{u_i} = 1 + \lfloor \frac{n}{2} \rfloor = \lceil \frac{n+1}{2} \rceil$. From symmetry all the eccentricities of the vertices of U equals the same value. Therefore the proof is completed. □

Proposition 3.2. *The eccentricitiy of a vertex w_i of W in $GP(n,1)$ is $\varepsilon_{w_i} = \lceil \frac{n+1}{2} \rceil$.*

Proof. Let $w_i \in V$. From the same arguments of the proof of the Proposition **??**, we get the eccentric paths of w_i,

$$w_i u_i u_{i+1} u_{i+2} ... u_{i+\lfloor \frac{n}{2} \rfloor - 1} u_{i+\lfloor \frac{n}{2} \rfloor}$$

or $w_i w_{i+1} w_{i+2} ... w_{i+\lfloor \frac{n}{2} \rfloor - 1} w_{i+\lfloor \frac{n}{2} \rfloor} u_{i+\lfloor \frac{n}{2} \rfloor}$. Thus $\varepsilon_{w_i} = 1 + \lfloor \frac{n}{2} \rfloor = \lceil \frac{n+1}{2} \rceil$. From symmetry, all the eccentricities of the vertices of W equal the same value. Therefore the proof is completed. □

Proposition 3.3. *Let $n \geqslant 3$ be odd integer. The eccentricitiy of a vertex u_i of U in $GP(n,2)$ is $\varepsilon_{u_i} = \lfloor \frac{n}{2} \rfloor$.*

Proof. Let u_i be a vertex of U. Then the paths

$$u_i u_{i+1} ... u_{i+\lfloor \frac{n}{2} \rfloor}$$

and

$$u_i w_i w_{i+2} ... w_{i+\lfloor \frac{n}{2} \rfloor} u_{i+\lfloor \frac{n}{2} \rfloor}$$

are the eccentric paths of the vertex u_i. Then the eccentricity of u_i is $\lfloor \frac{n}{2} \rfloor$. From the symmetry all the eccentricities of the vertices of U are $\lfloor \frac{n}{2} \rfloor$. □

Proposition 3.4. *Let $n \geqslant 6$ be even integer. The eccentricitiy of a vertex u_i of U in $GP(n,2)$ is $\varepsilon_{u_i} = \lfloor \frac{n}{2} \rfloor$.*

Proof. Let u_i be a vertex of U. Then the paths

$$u_i u_{i+1} ... u_{i+\lfloor \frac{n}{2} \rfloor}$$

and

$$u_i u_{i+1} w_{i+1} w_{i+3} ... w_{i+\lfloor \frac{n}{2} \rfloor} u_{i+\lfloor \frac{n}{2} \rfloor}$$

are the eccentric paths of the vertex

$$u_i$$

. Then the eccentricity of u_i is $\lfloor \frac{n}{2} \rfloor$. From the symmetry all the eccentricities of the vertices of U are $\lfloor \frac{n}{2} \rfloor$. □

Proposition 3.5. *Let $n \geqslant 3$ be odd integer. The eccentricitiy of a vertex w_i of W in $GP(n,2)$ is $\varepsilon_{w_i} = \lfloor \frac{n}{2} \rfloor$.*

Proof. Let $w_i \in V$. The path

$$w_i u_i u_{i+1} u_{i+2} ... u_{i+\lfloor \frac{n}{2} \rfloor - 1} u_{i+\lfloor \frac{n}{2} \rfloor}$$

is not the eccentric path of w_i. Because the path $w_i w_{i+2} ... w_{i+\lfloor \frac{n}{2} \rfloor} u_{i+\lfloor \frac{n}{2} \rfloor}$ is shorter. Then the eccentricity of w_i is $\lfloor \frac{n}{2} \rfloor$. From the symmetry all the eccentricities of the vertices of W are $\lfloor \frac{n}{2} \rfloor$. □

Proposition 3.6. *Let $n \geqslant 6$ be even integer. The eccentricitiy of a vertex w_i of W in $GP(n,2)$ is $\varepsilon_{w_i} = \lfloor \frac{n}{2} \rfloor$.*

Proof. Let $w_i \in V$. The path

$$w_i u_i u_{i+1} u_{i+2} ... u_{i+\lfloor \frac{n}{2} \rfloor -1} u_{i+\lfloor \frac{n}{2} \rfloor} w_{i+\lfloor \frac{n}{2} \rfloor}$$

is not the eccentric path of w_i. Because the path

$$w_i u_i u_{i+1} u_{i+2} w_{i+2} ... w_{i+\lfloor \frac{n}{2} \rfloor}$$

is shorter. Then the eccentricity of w_i is $\lfloor \frac{n}{2} \rfloor$. From the symmetry all the eccentricities of the vertices of W are $\lfloor \frac{n}{2} \rfloor$. □

Theorem 3.7. *The eccentric connectivity index of* $GP(n,1)$ *is* $\varepsilon^c(GP(n,1)) = 6n \lceil \frac{n+1}{2} \rceil$..

Proof. From the definition of the eccentric connectivity index, we can directly write;

$$\varepsilon^c(G) = \varepsilon^c(GP(n,1)) = \sum_{v \in V} \varepsilon_v d_v.$$

. Since the all generalized Petersen graphs are 3-regular, then we get;

$$\varepsilon^c(GP(n,1)) = \sum_{v \in V} \varepsilon_v d_v = 3 \sum_{v \in V} \varepsilon_v.$$

We know that

$$\varepsilon_v = \left\lceil \frac{n+1}{2} \right\rceil$$

for every vertex of $v \in V$ from Proposition 3.1 and Proposition 3.2. And we know that the generalized Petersen graphs have $2n$ vertices from the definition of the generalized Petersen graphs. Then, we can get;

$$\varepsilon^c(GP(n,1)) = 3 \sum_{v \in V} \varepsilon_v = 3.2n \left\lceil \frac{n+1}{2} \right\rceil = 6n \left\lceil \frac{n+1}{2} \right\rceil$$

. □

Theorem 3.8. *The eccentric connectivity index of* $GP(n,2)$ *is* $\varepsilon^c(GP(n,2)) = 6n \lfloor \frac{n}{2} \rfloor$..

Proof. From the same facts stated in the proof of Theorem 3.7 we can write;

$$\varepsilon^c(GP(n,2)) = \sum_{v \in V} \varepsilon_v d_v = 3 \sum_{v \in V} \varepsilon_v$$

. We know that

$$\varepsilon_v = \left\lfloor \frac{n}{2} \right\rfloor$$

from the Proposition 3.3 , Proposition 3.4, Proposition 3.5 and Proposition 3.6. Then, we can get;

$$\varepsilon^c(GP(n,2)) = 3 \sum_{v \in V} \varepsilon_v = 6n \left\lfloor \frac{n}{2} \right\rfloor.$$

□

Theorem 3.9. *The connective eccentricity index of* $GP(n,1)$ *is* $\varepsilon^{ce}(GP(n,1)) = \frac{6n}{\lceil \frac{n+1}{2} \rceil}$.

Proof. We can directly write $\varepsilon^{ce}(GP(n,1)) = \sum_{v \in V} \frac{d_v}{\varepsilon_v}$. From the above arguments in Theorem 3.7 and the definition of the connective eccentricity index, we get that; $\varepsilon^{ce}(G) = \varepsilon^{ce}(GP(n,1)) = \sum_{v \in V} \frac{d_v}{\varepsilon_v} = 3.2n. \frac{1}{\lceil \frac{n+1}{2} \rceil} = \frac{6n}{\lceil \frac{n+1}{2} \rceil}$. □

Theorem 3.10. *The connective eccentricity index of* $GP(n,2)$ *is* $\varepsilon^{ce}\left(GP\left(n,2\right)\right)=\frac{6n}{\left\lfloor\frac{n}{2}\right\rfloor}$.

Proof. We can directly write $\varepsilon^{ce}\left(GP\left(n,1\right)\right)=\sum\limits_{v\in V}\frac{d_v}{\varepsilon_v}$. From the above arguments in Theorem 3.8 and the definition of the connective eccentricity index, we get that; $\varepsilon^{ce}(G)=\varepsilon^{ce}(GP(n,2))=\sum\limits_{v\in V}\frac{d_v}{\varepsilon_v}=3.2n.\frac{1}{\left\lfloor\frac{n}{2}\right\rfloor}=\frac{6n}{\left\lfloor\frac{n}{2}\right\rfloor}$. □

4. Conclusion

This study is the first study related to compute the exact value of certain topological indices of generalized Petersen graphs of $GP(n,1)$ and $GP(n,2)$ such as eccentric connectivity index and connective eccentricity index. It can be interesting to compute the other topological indices of generalized Petersen graphs for further studies such as adjacent eccentric distance sum index, eccentric distance sum, Wiener group indices, reverse Zagreb indices, Hosoya index, PI index and some other degree and distance based topological indices. It can also be interesting to compute the exact value of eccentric connectivity index and connective eccentricity index of generalized Petersen graphs $GP(n,3)$ and $GP(n,k)$ for further studies.

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